Methods Masterclass

QCE Mathematical Methods Units 3 and 4

Wednesday 11 November, 3 to 6:30pm · UQ St Lucia 24-S304 and Zoom

Student workbook

24 exam-style questions in syllabus order, Unit 3 then Unit 4. Questions only: tutors work the solutions live.

- Level 1 = simple familiar, Level 2 = complex familiar, Level 3 = complex unfamiliar.
- Tech-free questions are Paper 1 style. Tech-active questions are Paper 2 style.
- Try every question on your own first. Write the set-up before you calculate.
- Paper 1 (tech-free) Friday 13 Nov, AM. Paper 2 (tech-active) Friday 13 Nov, PM.

This is not an official QCAA publication. Questions are original and written in the style of QCAA external assessment.

The plan for today

Time	Block	Tutor	Questions	Pages
3:00	Welcome and how today runs	Rishabh		
3:05	**Exponentials, logs and derivatives**	Rishabh	Q1 to Q4	3 to 7
3:30	**Applications of differentiation**	Julian	Q5 to Q8	8 to 12
4:00	Break / Blooket			
4:10	**Integration**	Senula	Q9 to Q11	13 to 16
4:35	**Trigonometry: triangles and 3D**	Julian	Q12 to Q15	17 to 21
5:05	**Discrete probability and binomial**	Rishabh	Q16 to Q18	22 to 25
5:30	Break			
5:40	**Continuous random variables and the normal**	Senula + Julian	Q19 to Q21	26 to 29
6:00	**Sampling, confidence intervals and complex unfamiliar**	Senula + Rishabh	Q22 to Q24	30 to 33
6:25	You're ready. Questions and close	Rishabh		

Your three goals today

- Write the set-up for every question before touching a calculator.
- Circle one step you missed in each block. That is tonight's revision.
- Learn the QCAA sentences: concavity changes, binomial conditions, what 95% confidence means.

Block 1 · Exponentials, logs and derivatives

3:05 to 3:30 · Rishabh · Unit 3

Three key ideas

1. Hidden quadratics. $4^x = (2^x)^2$. Let $u = 2^x$, solve the quadratic, then reject any $u \leq 0$.

2. Log laws, then check. Combine logs into one, convert to index form, then substitute every answer back. Logs of negatives are undefined.

3. Chain, product, quotient. Name the rule before you start. e^{kx} gives $k\,e^{kx}$. $\ln(f(x))$ gives $f'(x)/f(x)$.

Formulas to know

Log laws	$\log(ab) = \log a + \log b, \quad \log(a/b) = \log a - \log b, \quad \log(a^n) = n \log a$
Index form	$\log_a(y) = x \Leftrightarrow a^x = y$
Exponential	$d/dx\ e^{kx} = k\,e^{kx}$
Log	$d/dx\ \ln(f(x)) = f'(x) / f(x)$
Trig	$d/dx\ \sin(kx) = k\cos(kx), \quad d/dx\ \cos(kx) = -k\sin(kx)$
Product	$(uv)' = u'v + uv'$
Chain	$dy/dx = dy/du \times du/dx$

Traps to avoid

- × Writing $\log(a + b) = \log a + \log b$
- × Forgetting to reject $u = 2^x \leq 0$
- × Not checking log answers by substitution (QCAA 2025 flagged this)
- × Dropping the inner derivative in the chain rule
- × Calculator in degrees for calculus questions

Question 1 Hidden quadratic

Level 1 · 3 marks · tech-free · 4 min

Solve the equation below for x.

$4^x - 3 \cdot 2^{x+1} + 8 = 0$ [3]

Question 2 Logs that lead to a quadratic

Level 2 · 4 marks · tech-free · 6 min

Consider the equation $\log_3(x + 2) + \log_3(x - 4) = 3$.

Solve the equation for x. Justify the rejection of any solution and verify your answer. [4]

Question 3 Derivative toolkit

Level 1 · 2 marks · tech-free · 3 min

Let $y = x^2 e^{3x}$.

Determine dy/dx. [2]

Question 4 Drug concentration model

Level 3 · 4 marks · tech-active · 8 min

The concentration of a drug in a patient's blood is modelled by $C(t) = A\,e^{-kt}$ mg L^{-1}, where t is the time in hours after the injection. Measurements show $C(1) = 48$ and $C(3) = 27$.

The drug is no longer effective once the concentration falls below 8 mg L^{-1}. Determine how long after the injection this occurs, correct to the nearest minute. [4]

Block 2 · Applications of differentiation

3:30 to 4:00 · Julian · Unit 3

Three key ideas

1. Stationary points. Solve f′(x) = 0, then use f″ to classify: negative means maximum, positive means minimum.

2. Inflection needs a change. f″ = 0 is not enough. Show f″ changes sign either side (QCAA 2025: most students lost this mark).

3. Optimisation recipe. Write the quantity in ONE variable, differentiate, set to zero, justify max or min, answer the question asked.

Formulas to know

Stationary point	$f'(x) = 0$
Maximum	$f'(a) = 0$ and $f''(a) < 0$
Minimum	$f'(a) = 0$ and $f''(a) > 0$
Inflection	$f''(a) = 0$ AND f'' changes sign at $x = a$
Motion	$x \to v = dx/dt \to a = dv/dt$
Back again	$x = \int v\ dt, \quad v = \int a\ dt$ (use the given values to find c)

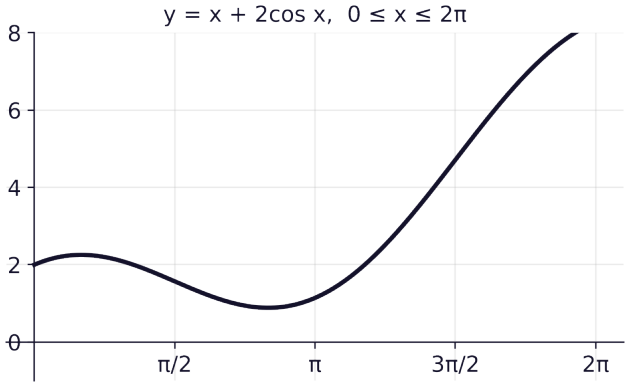
Traps to avoid

- × Stopping at f″ = 0 for an inflection point
- × Giving θ when the question asks for the maximum AREA
- × Not stating the domain or checking endpoints
- × Mixing up displacement and distance
- × Forgetting the + c when going back from v to x

Question 5 Stationary points and inflection

Level 2 · 6 marks · tech-free · 9 min

Let f(x) = x + 2 cos x for 0 ≤ x ≤ 2π.

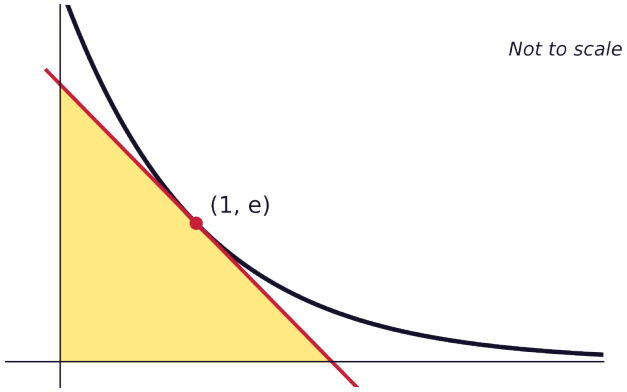


Determine the x-coordinates and nature of the stationary points of f, and show that the graph of f has a point of inflection at x = π/2. [6]

Question 6 Tangent triangle

Level 2 · 4 marks · tech-free · 6 min

The tangent to the graph of y = e^(2 − x) at the point (1, e) forms a triangle with the coordinate axes, as shown (not to scale).

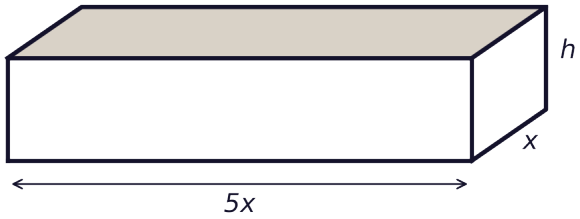


Determine the exact area of this triangle. [4]

Question 7 Brick optimisation

Level 3 · 6 marks · tech-free · 10 min

A solid brick in the shape of a cuboid measures 5x cm long, x cm wide and h cm high. The total surface area of the brick is 720 cm².



Determine the exact maximum volume of the brick, and justify that it is a maximum. [6]

Question 8 Motion with two unknowns

Level 3 · 5 marks · tech-free · 9 min

A particle moves in a straight line with velocity $v(t) = p\cos(qt)$ m s^{-1}, where p and q are positive constants and $t \geq 0$ is in seconds. Its initial velocity is 6 m s^{-1} and it first comes to rest at $t = \pi$ s. Its initial displacement is 0 m.

Determine the total distance travelled by the particle in the first 2π seconds. [5]

Block 3 · Integration

4:10 to 4:35 · Senula · Units 3 and 4

Three key ideas

1. Reverse the chain rule. $\int e^{kx}\,dx = (1/k)\,e^{kx} + c$. Divide by the inside coefficient.

2. Area between curves. Find the intersections, decide which curve is on top, then integrate top minus bottom.

3. Area is never negative. If a region crosses the x-axis, split it. A signed integral can cancel to zero.

Formulas to know

Exponential	$\int e^{kx}\,dx = (1/k)\,e^{kx} + c$
Reciprocal	$\int 1/x\,dx = \ln\|x\| + c$
Trig	$\int \sin(kx)\,dx = -(1/k)\cos(kx) + c, \quad \int \cos(kx)\,dx = (1/k)\sin(kx) + c$
Definite	$\int \text{from a to b of } f(x)\,dx = F(b) - F(a)$
Between curves	$\text{Area} = \int (\text{top} - \text{bottom})\,dx$
Tangent line	$y - f(a) = f'(a)(x - a)$

Traps to avoid

- × Multiplying by k instead of dividing
- × Bottom minus top gives a negative area
- × Missing an intersection point
- × Writing ln x instead of ln|x| in indefinite integrals
- × Forgetting the exact form (leave ln 2, e, π)

Question 9 Integration warm-up

Level 1 · 2 marks · tech-free · 3 min

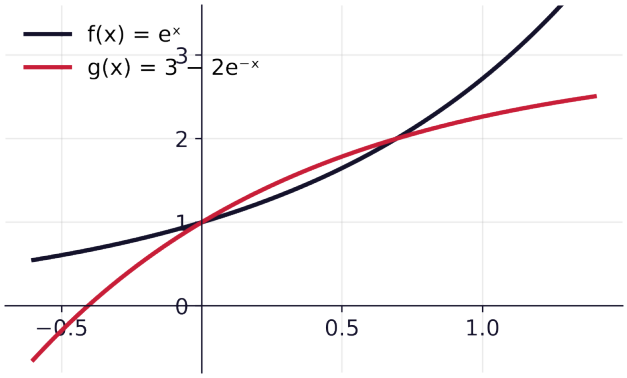
Evaluate the definite integral below.

$\int_0^{\pi/4} 8\cos(2x)\,dx$ [2]

Question 10 Area between two curves

Level 2 · 5 marks · tech-free · 8 min

The graphs of $f(x) = e^x$ and $g(x) = 3 - 2e^{-x}$ are shown.

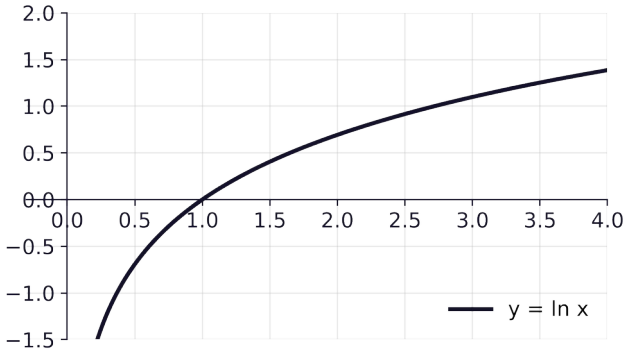


Determine the exact area of the region enclosed by the graphs of f and g. [5]

Question 11 Tangent to ln x

Level 3 · 5 marks · tech-free · 12 min

The straight line $y = mx$, where $m > 0$, is a tangent to the curve $y = \ln x$.



Given that $\frac{d}{dx}(x \ln x - x) = \ln x$, determine the exact area of the region bounded by the tangent, the curve $y = \ln x$ and the x-axis. [5]

Block 4 · Trigonometry: triangles and 3D

4:35 to 5:05 · Julian · Unit 4

Three key ideas

1. Pick the rule from what you have. Right angle: SOH CAH TOA. Two angles and a side, or a side-angle pair: sine rule. Three sides, or two sides and the angle between: cosine rule.

2. Unit circle and graphs. sin is the y-coordinate, cos is the x-coordinate. For y = a sin(b(x − c)) + d: amplitude a, period 2π/b, phase shift c, median line d.

3. 3D means two flat triangles. Draw the vertical triangle and the horizontal (ground) triangle separately, then link them through a shared side.

Formulas to know

Sine rule	$a / \sin A = b / \sin B = c / \sin C$
Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$
Angle form	$\cos C = (a^2 + b^2 - c^2) / (2ab)$
Area	$\text{Area} = ½ ab \sin C$
Exact values	$\sin 30° = ½,\ \cos 30° = \sqrt{3}/2,\ \sin 150° = ½,\ \cos 150° = -\sqrt{3}/2$
Elevation	$\tan(\text{angle of elevation}) = \text{height} / \text{horizontal distance}$

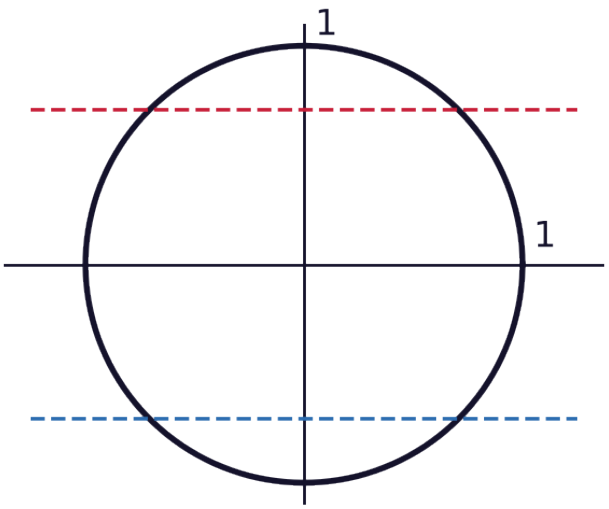
Traps to avoid

- × Calculator in radians for a degrees question (the opposite of calculus!)
- × Missing the second angle in the ambiguous case
- × Using the cosine rule with an angle that is not between the two sides
- × Mixing up angle of elevation and depression
- × Drawing a 3D question as one flat triangle

Question 12 Unit circle equation

Level 1 · 3 marks · tech-free · 4 min

Consider the equation $2\sin^2\theta - 1 = 0$.

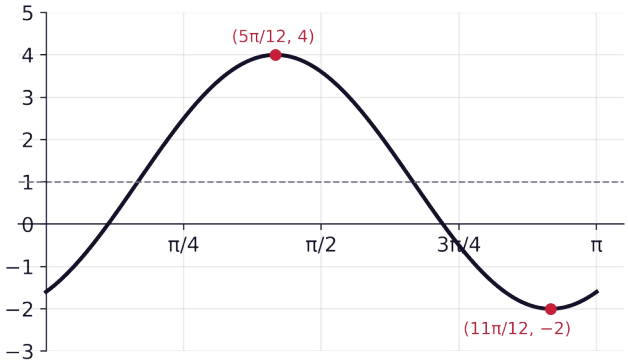


Solve the equation for $0 \leq \theta \leq 2\pi$. [3]

Question 13 Reading a trig graph

Level 2 · 4 marks · tech-free · 6 min

The graph of $y = a\sin(b(x - c)) + d$, where $a > 0$, $b > 0$ and $0 < c < \pi/2$, is shown. It has a maximum at $(5\pi/12, 4)$ and a minimum at $(11\pi/12, -2)$.



Determine the values of a, b, c and d. [4]

Question 14 Garden bed

Level 1 · 4 marks · tech-free · 5 min

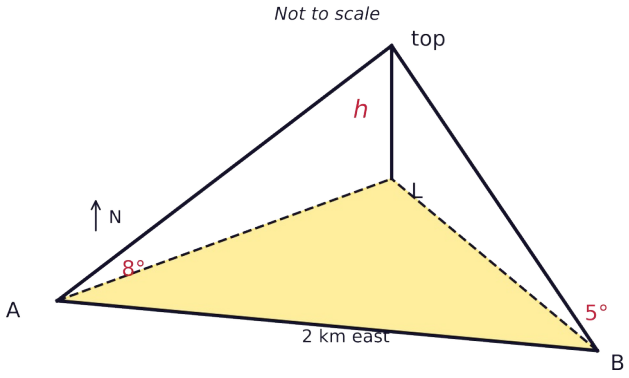
A triangular garden bed has two sides of length 8 m and 5 m, with an angle of 150° between them.

Determine the exact area of the garden bed and the exact length of its third side. [4]

Question 15 Lighthouse from two boats

Level 3 · 5 marks · tech-active · 10 min

A boat at A is due south of a lighthouse L. From A, the angle of elevation of the top of the lighthouse is 8°. The boat then sails 2 km due east to B, where the angle of elevation of the top is 5°.



Determine the height of the lighthouse, correct to the nearest metre. [5]

Block 5 · Discrete probability and binomial

5:05 to 5:30 · Rishabh · Unit 3

Three key ideas

1. Probabilities add to 1. Use ΣP = 1 to find unknowns. E(X) = Σ x P(x).

2. Is it binomial?. Fixed n, two outcomes, constant p, independent trials. Say all four in words when asked.

3. Conditional = overlap / given. P(A | B) = P(A ∩ B) / P(B). Underline the word 'given' in the question.

Formulas to know

Expected value	$\mathrm{E}(\mathrm{X}) = \Sigma\ \mathrm{x}\cdot\mathrm{P}(\mathrm{X} = \mathrm{x})$
Variance	$\mathrm{Var}(\mathrm{X}) = \mathrm{E}(\mathrm{X}^2) - [\mathrm{E}(\mathrm{X})]^2$
Binomial	$\mathrm{P}(\mathrm{X} = \mathrm{x}) = \mathrm{C}(\mathrm{n},\ \mathrm{x})\ \mathrm{p}^{\mathrm{x}}\ (1 - \mathrm{p})^{\mathrm{n-x}}$
Binomial mean and variance	$\mathrm{E}(\mathrm{X}) = \mathrm{np},\quad \mathrm{Var}(\mathrm{X}) = \mathrm{np}(1 - \mathrm{p})$
Conditional	$\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \mathrm{P}(\mathrm{A} \cap \mathrm{B})\ /\ \mathrm{P}(\mathrm{B})$
Total probability	$\mathrm{P}(\mathrm{B}) = \Sigma\ \mathrm{P}(\mathrm{B} \mid \mathrm{A_i})\ \mathrm{P}(\mathrm{A_i})$

Traps to avoid

- × Var(X) = E(X²) − E(X), forgetting to square
- × 'At least 2' means 1 − P(X ≤ 1), not 1 − P(X ≤ 2)
- × Using binomcdf when the question wants exactly one value
- × Dividing by the wrong thing in conditional probability
- × Rounding too early in multi-step questions

Question 16 Discrete random variable

Level 1 · 4 marks · tech-free · 5 min

A discrete random variable X has $P(X = x) = k(5 - x)$ for $x \in \{1, 2, 3, 4\}$.

a) Show that $k = 0.1$. [1]

b) Determine $E(X)$. [1]

c) Determine $\text{Var}(X)$. [2]

Question 17 Faulty bulbs

Level 2 · 4 marks · tech-active · 7 min

Each light bulb in a box of 20 is faulty with probability 0.08, independently of the others. Let X be the number of faulty bulbs in a box.

A box is found to contain at least one faulty bulb. Determine the probability that it contains exactly 2 faulty bulbs. [4]

Question 18 Late to school

Level 2 · 4 marks · tech-free · 6 min

Students travel to school by train (45%), bus (35%) or on foot (20%). The probability that a student is late is 0.08 for train, 0.12 for bus and 0.05 for walkers.

A randomly chosen student is late. Determine the probability that they caught the bus. [4]

Block 6 · Continuous random variables and the normal

5:40 to 6:00 · Senula + Julian · Unit 4

Three key ideas

1. A PDF is an area machine. Total area = 1. Probability = area under f between two values. P(X = a) = 0.

2. Mean and variance by integrals. E(X) = ∫ x f(x) dx. Var(X) = ∫ x² f(x) dx − μ².

3. Normal: standardise or use the calculator. z = (x − μ)/σ. Use inverse normal to work backwards from a probability.

Formulas to know

Total area	$\int f(x)\,dx$ over the domain $= 1$
Probability	$P(a < X < b) = \int$ from a to b of $f(x)\,dx$
Mean	$E(X) = \int x\,f(x)\,dx$
Variance	$\mathrm{Var}(X) = E(X^2) - \mu^2$
CDF	$F(x) = P(X \le x) = \int$ from min to x of $f(t)\,dt$
Standardise	$z = (x - \mu) / \sigma$

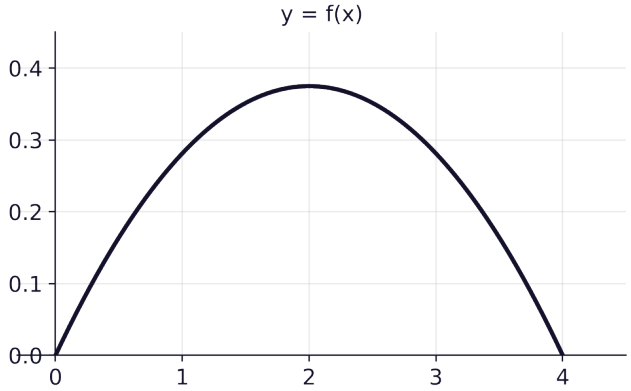
Traps to avoid

- × Forgetting f(x) is 0 outside the domain
- × Using σ² where the calculator wants σ
- × Inverse normal with the wrong tail
- × Treating the PDF value as a probability
- × Not giving units (g, minutes) in the final answer

Question 19 Probability density function

Level 2 · 6 marks · tech-active · 9 min

A continuous random variable X has probability density function f(x) = kx(4 − x) for 0 ≤ x ≤ 4, and f(x) = 0 elsewhere.



a) Show that k = 3/32. [2]

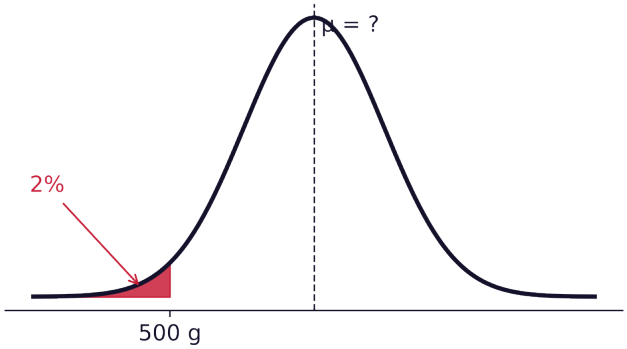
b) Determine P(X > 3). [2]

c) Determine Var(X). [2]

Question 20 Packet masses

Level 2 · 4 marks · tech-active · 6 min

The mass of a packet of rice is normally distributed with mean 502 g and standard deviation 3 g.



a) Determine the probability that a packet has a mass under 495 g. [1]

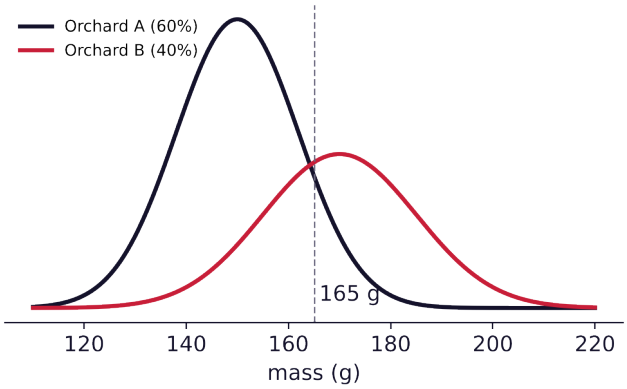
b) Determine P(498 < mass < 505). [1]

c) The machine is adjusted so that only 2% of packets are under 500 g. The standard deviation is unchanged. Determine the new mean, correct to one decimal place. [2]

Question 21 Two orchards

Level 3 · 5 marks · tech-active · 9 min

Apples from orchard A have masses that are normally distributed with mean 150 g and standard deviation 12 g. Apples from orchard B have masses that are normally distributed with mean 170 g and standard deviation 15 g. A crate contains 60% orchard A apples and 40% orchard B apples.



An apple chosen at random from the crate has a mass greater than 165 g. Determine the probability that it came from orchard B. [5]

Block 7 · Sampling, confidence intervals and complex unfamiliar

6:00 to 6:25 · Senula + Rishabh · Unit 4

Three key ideas

1. p̂ is a random variable. p̂ = x/n changes from sample to sample. Its spread shrinks as n grows: SE = √(p̂(1 − p̂)/n).

2. Build the interval. p̂ ± z·SE. Use z = 1.96 for 95%. Tech-free: the numbers are chosen so the square root is nice.

3. Use the interval to judge a claim. If the claimed value is outside the interval, the data does not support it. Inside means consistent, not proven.

Formulas to know

Sample proportion	$\hat{p} = x \,/\, n$
Standard error	$SE = \sqrt{\hat{p}(1-\hat{p}) \,/\, n}$
Interval	$\hat{p} - z{\cdot}SE \;,\; \hat{p} + z{\cdot}SE$
z values	90%: 1.645 95%: 1.96 99%: 2.576
Margin of error	$E = z{\cdot}SE \quad (\text{width} = 2E)$
More confidence	wider interval; bigger n: narrower interval

Traps to avoid

- × Saying 'there is a 95% chance p is in this interval'
- × Using a percentage (20) instead of a proportion (0.2)
- × Saying a claim is 'proven' because it is inside the interval
- × Not answering the claim in context
- × Forgetting to define the binomial variable in CU questions

Question 22 Working backwards from an interval

Level 3 · 6 marks · tech-free · 12 min

A random sample of people were asked whether they support a new bike lane. Using $z = 1.96$, an approximate 95% confidence interval for the proportion who support it was calculated to be (0.201, 0.299).

Determine how many people were surveyed, how many supported the bike lane, and the sample size that would be needed to halve the margin of error, assuming the same sample proportion. [6]

Question 23 Defective bolts

Level 3 · 6 marks · tech-active · 12 min

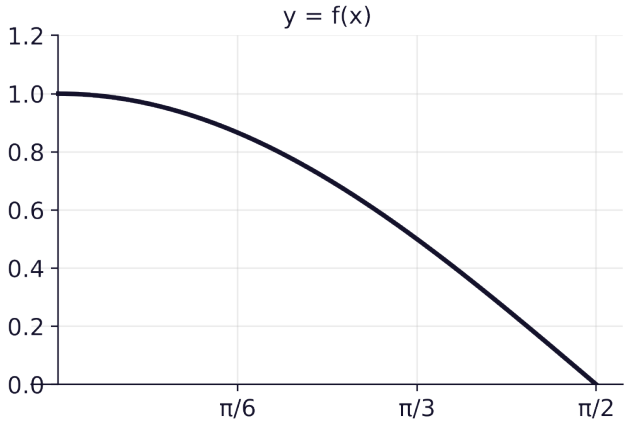
A machine makes bolts whose diameters are normally distributed with mean 10 mm and standard deviation 0.06 mm. A bolt is defective if its diameter is less than 9.9 mm. Bolts are sold in boxes of 20.

The manufacturer wants at most 20% of boxes to contain at least one defective bolt. The mean cannot be changed, but the standard deviation can be reduced by adjusting the machine. Determine the standard deviation required, correct to three decimal places. [6]

Question 24 Final boss: trig meets probability

Level 3 · 5 marks · tech-free · 10 min

A continuous random variable X has probability density function f(x) = k cos x for 0 ≤ x ≤ π/2, and f(x) = 0 elsewhere.



Determine the value of k, the exact median of X, and the exact value of P(X > π/3). [5]

Question B1 Tide height

Level 2 · 2 marks · tech-active · 7 min

The depth of water at a jetty is $h(t) = 4 + 1.5\sin(\pi t/6)$ m, where t is hours after midnight.

A boat needs at least 4.5 m of water. Determine for how long in one 12-hour cycle the boat can use the jetty. [2]

Question B2 Majority claim

Level 2 · 5 marks · tech-active · 7 min

In a random sample of 250 voters, 118 support option A.

A campaigner claims that a majority of voters support option A. Use a 95% confidence interval to evaluate this claim. [5]